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Technical Comments

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Propulsion and Power *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Triggering of Longitudinal Combustion Instabilities in Rocket Motors: Nonlinear Combustion Response"

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Introduction

TRIGGERING of oscillations in rocket motors is the subject of an ongoing investigation at the California Institute of Technology. Some issues that have been previously addressed in several recent publications by the authors[1–3] have more recently been examined by Wicker et al.[4] Whereas the latter paper provides a thorough investigation of the equations as derived, it apparently overlooks the possible effects of two key approximations that were used in the derivation of these equations. Some of the possible effects of these assumptions have been addressed previously,[3] and the authors suggest that the work by Wicker et al. might have benefited from our earlier and accessible conclusions.

Discussion

Wicker et al.[4] used two approximations that have been applied extensively in the past, namely, the method of time averaging and the truncation of the system to two modes. It was proposed that the two-mode, time-averaged expansion "captures the essential physics of the multiple degree of freedom, nonlinear system.[4]" This statement, based on work dealing exclusively with spontaneous oscillations (supercritical bifurcations),[5] should not be applied without further investigation to systems that exhibit the qualitatively different phenomenon of triggering (subcritical bifurcations). The task of determining the effects of these approximations, however, is not always easy. The reason for using these approximations in the first place is to simplify the system to the point that it can be solved analytically; the more complex original system of equations must be solved to check the validity of the approximations. Much of our recent work has dealt with solving the original acoustic equations so that the effects of these assumptions can be determined. To accomplish this, we have applied the methods of dynamic systems theory to both the original equations and the time-averaged equations. In particular, a continuation method was used to determine the steady states and periodic solutions of the system; see Jahnke and Culick[5] and Burnley[3] for details. In their study, Wicker et al. found the possibility of triggering to stable limit cycles when using a nonlinear combustion response related to the magnitude of velocity parallel to the burning surface, that is, $\dot{m}'_b \propto |u'|$. Similar results were reported in the studies by Kim[6] and Greene.[7] However, all of these investigations truncated the expansion to two modes without examining the possible consequences. By including four and six modes in the expansion, we have studied the possible effects of truncation to a small number of modes when using this type of nonlinear combustion response.[3] As seen in Fig. 1, values of the nonlinear combustion response function, $\bar{R}_{vc}$, which in this case produce a subcritical bifurcation for the two-mode expansion, do not necessarily produce subcritical bifurcations when more than two modes are included. In the absence of contrary evidence, that result raises questions regarding the validity of the results reported by Wicker et al. We feel that these issues warrant further investigation, and a more thorough study is currently in preparation by the authors.

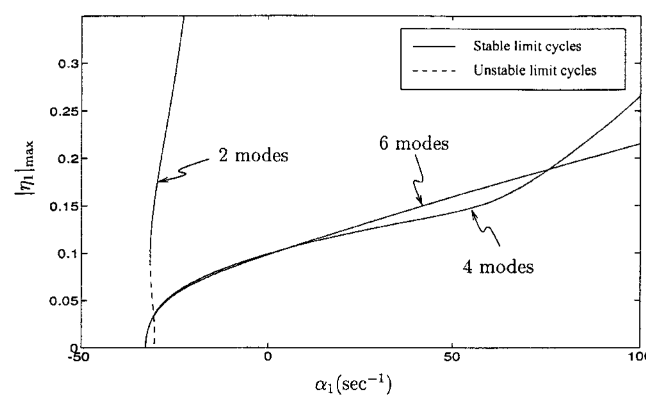

Fig. 1 Example showing the effect of truncation to a small number of modes; time-averaged equations with Greene's nonlinear combustion model, $\bar{R}_{vc}$ = 5.32.

It has been our experience that the nonlinear combustion response introduced by Greene[7] and used by Wicker et al.[4] is generally inefficient at the type of coupling required to produce triggering. As a result, a new model based on the experimental results of Ma et al.[8] was introduced in our investigations. This model, based on the idea of a threshold velocity, appears to be more efficient at producing true nonlinear instabilities.[2]

Summary

The approximations of time-averaging and truncation to two modes have been extremely useful in the study of spontaneous instabilities in combustion chambers. When applying these approximations to the qualitatively different phenomenon of triggering, however, care should be taken to determine the possible effects of applying these approximations. Some of these consequences have been treated previously and further work is underway to understand better the complex phenomenon of triggering.

References

[1] Culick, F. E. C., Burnley, V. S., and Swenson, G., "Pulsed Instabilities in Solid-Propellant Rockets," *Journal of Propulsion and Power*, Vol. 11, No. 4, 1995, pp. 657–665.

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[2]Burnley, V. S., Swenson, G., and Culick, F. E. C., "Pulsed Instabilities in Combustion Chambers," *31st AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, San Diego, CA, AIAA, Washington, DC, 1995; also AIAA Paper 95-2430, 1995.

[3]Burnley, V. S., "Nonlinear Combustion Instabilities and Stochastic Sources," Ph.D. Dissertation, California Inst. of Technology, Pasadena, CA, 1996.

[4]Wicker, J. M., Greene, W. D., Kim, S.-I., and Yang, V., "Triggering of Longitudinal Combustion Instabilities in Rocket Motors: Nonlinear Combustion Response," *Journal of Propulsion and Power*, Vol. 12, No. 6, 1996, pp. 1148–1158.

[5]Jahnke, C. C., and Culick, F. E. C., "An Application of Dynamical Systems Theory to Nonlinear Combustion Instabilities," *Journal of Propulsion and Power*, Vol. 10, No. 4, 1994, pp. 508–517.

[6]Kim, S. I., "Nonlinear Combustion Instabilities in Combustion Chambers," Ph.D. Dissertation, Dept. of Mechanical Engineering, Pennsylvania State Univ., University Park, PA, May 1989.

[7]Greene, W. D., "Triggering of Longitudinal Combustion Instabilities in Rocket Motors," M.S. Thesis, Dept. of Aerospace Engineering, Pennsylvania State Univ., University Park, PA, Dec. 1990.

[8]Ma, Y., Van Moorhem, W. K., and Shorthill, R. W., "Experimental Investigation of Velocity Coupling in Combustion Instability," *Journal of Propulsion and Power*, Vol. 7, No. 5, 1991, pp. 692–699.

Reply by the Authors to V. S. Burnley and F. E. C. Culick

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Introduction

IN Ref. 1, various aspects of the problem of triggered instabilities were addressed using an analytical approach. In Ref. 2, a Comment on Ref. 1, by Burnley and Culick, two issues are raised questioning the validity of this analytical approach. The presentation of the first issue is somewhat confusing because it appears to be resolved by the authors within the comment itself. It is suggested that applying the method of time averaging would have a significant effect on the qualitative behavior of the system, but results are then explicitly cited that indicate that this proposition is erroneous.[3] The second issue raised in the Comment involves the two-mode expansion. Results are cited that suggest that the pulse-triggered behavior found using the two-mode approximation is merely an artifact of that approximation and is not physical.[2] However, these results appear to be incapable of supporting such a general conclusion by themselves, and the method on which the results are based seems to be limited to parametric-type investigations. The two-mode expansion is used in Ref. 1 to allow analytical tractability and to provide physical insight into the mechanism involved in triggered instabilities, such that conclusions of broad scope can be drawn.

Response

The details of our response to the Comment are given in the following three parts.

First, in their Comment on Ref. 1, Burnley and Culick[2] cite results based on the continuation method. This is a technique used in dynamic systems theory for the analysis of systems of ordinary differential equations.[4] The method deals specifically with the steady states of a system, those states for which all of the time derivatives of the system are equal to zero. According to the Hartman–Grobman theorem, the local stability of any steady state can be determined by the stability of the system linearized about that steady-state point. Also, the implicit function theorem provides that the steady states of a system are continuous functions of the system's parameters. The continuation method employs these two theorems to efficiently investigate the behavior of a dynamic system in terms of the system parameters (avoiding explicit solution of the differential system itself). Of particular interest in Ref. 2 are bifurcations, that is, points where qualitative changes in system behavior occur as a parameter (or parameters) is changed. In Ref. 2, the equations governing the flow oscillations are treated with a Galerkin-based method (similar to Ref. 1), resulting in a set of ordinary differential equations that govern the time behavior of the coefficients in the series of acoustic modes used to represent the unsteady motions. In their derivation, the combustion response was represented in the forms from Greene[5] and Kim[6] along with a form due to Baum et al.[7] and Levine and Baum.[8] Upon application of the continuation method for a limited set of system parameters, scenarios were presented in which triggered behavior was observed for two modes, but not for a many-mode, for example, six-mode, model.

Second, the primary concern raised in the Comment regards the two-mode expansion. To apply the Galerkin-based method as in Ref. 1, the unsteady pressure and velocity fields are expressed as an infinite series of the acoustic normal modes. This expansion allows the continuum dynamics of the unsteady flowfield to be treated as a system with discrete states, that is the time-varying mode amplitude coefficients. The resulting discrete system has infinite degrees of freedom, and so, practically, the series must be truncated to a finite number of modes. In Ref. 1, two modes were retained mainly to preserve analytical tractability. This allows global conclusions to be drawn about the behavior of the system, as has been demonstrated in many previous works on nonlinear aspects of combustion instability. For instance, it was possible to rule out several types of combustion response as mechanisms for triggering, allowing attention to be focused on the combustion response functions based on velocity rectification. Without the analytical tractability provided by the two-mode expansion, the analysis would be limited to parametric studies, from which it is often difficult or impossible to extract meaningful conclusions. The two-mode expansion also allows physical insight into the gasdynamic behavior, for example, in terms of mode coupling and intermodal energy transfer.

Third, the primary issue raised in the Comment is that the results in Ref. 1 are limited by the expansion of the nonlinear acoustic field to only the first two acoustic modes. It is suggested that the stable limit cycles found with the two-mode approximation are merely an artifact of that approximation and are not physical. In the Comment, a bifurcation diagram was given that illustrates the failure of the two-mode system in erroneously predicting triggering behavior. In this diagram, however, the first mode's linear growth rate α_1 was the only parameter that was varied. It seems implausible that global conclusions regarding the relevancy of the two-mode approximation could be made by examining the system characteristics only as a function of one parameter. The region of triggerability is likely an n-dimensional region in parametric space, where for the time-averaged, two-mode approximation n equals four (the parameters are the linear growth rate of the first and the second modes, phase difference parameter, and nonlinear combustion response parameter). For the non-time-averaged system with two modes, the linear parameters would form a 32-dimensional space (taking into account the diagonal and off-diagonal linear elements in the source term). Furthermore, this triggerable region may exist for any number of modes and may only be distorted by the two-mode approximation. Perhaps the set of α_1 values in the bifurcation diagram represents a line in the parameter space that is near the edge of the triggerable region; that is, it intersects the region for the two-mode system, but misses it entirely for more modes. Admittedly, this is speculative, and in Ref. 1 no attempt was made to investigate the effects of the two-mode expansion, mainly because the analytical tractability would be compromised. If no triggerable region exists throughout the entire parametric space for a many-mode expansion, then the failure of the two-mode system would be universal and not just

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